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MEMORANDUM REPORT ARCCB-MR-88014

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**A TECHNIQUE FOR MEASURING  
AREA UNDER A CURVE**

**J. H. UNDERWOOD**

MARCH 1988

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**US ARMY ARMAMENT RESEARCH,  
DEVELOPMENT AND ENGINEERING CENTER  
CLOSE COMBAT ARMAMENTS CENTER  
BENET LABORATORIES  
WATERVLIET, N.Y. 12189-4060**



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19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Area Measurement Mechanical Tests Fracture Toughness Plane Geometry		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A plane-geometry technique for measuring the area under a curve is described. Areas under mechanical load-deflection curves can be approximated by the area of a trapezoid. Accuracy of the technique is checked by applying it to a graphical construction of known area.		

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## INTRODUCTION

Measurement of area under a curve is often the most direct way to determine the energy of a mechanical system. Area measurements are now routinely used in fracture mechanics testing as part of the standard test method for the J-integral fracture toughness (ref 1). Vossoughi (ref 2) recently described a method for measuring the area under a curve, based on the weight of a template of the area. The technique suggested here is a simple geometric procedure suited to the relatively regular shapes of elastic-plastic load versus displacement plots of the type shown by curve OAB in Figure 1. This curve is typical of those obtained from tensile strength and fracture toughness tests of metals.

## DISCUSSION

The technique involves two straight lines. The first, OAC in the figure, is nearly always drawn as standard procedure in a load versus displacement test. It is fitted, usually by eye, to the linear elastic portion of the plot, ignoring any nonlinear behavior which may occur at low loads. The second line, CDE, is drawn horizontally so that area ACDA appears by eye to be equal to area DBED. Then the total area under the curve OAB can be accurately approximated by the area of a trapezoid:

$$\text{area under OAB} \approx \text{area under OCE} \quad (1)$$

and using plane geometry, this area,  $\alpha$ , is

$$\alpha = P_1 (V_{\max} - V_1/2) \quad (2)$$

---

<sup>1</sup>"Standard Test Method for  $J_{Ic}$ , A Measure of Fracture Toughness, ASTM E-813," Annual Book of ASTM Standards, Vol. 03.01, ASTM, Philadelphia, PA, 1987, pp. 968-990.

<sup>2</sup>Vossoughi, J., "A Simple, Quick, and Accurate Method to Measure the Area Under Curves," Experimental Techniques, Vol. 8, No. 10, 1984, pp. 26-27.

The calculation outlined by Eq. (2) can be used to measure area under a curve up to a point well beyond maximum load, typically to a point where the load has dropped by about 20 percent. Beyond this point, the horizontal line may intersect the curve again, and the calculation described here cannot be used without some modification.

#### CONCLUSION

A measure of the accuracy of the technique can be obtained by the following analysis of a graphical construction which was incorporated into Figure 1. The portion AB of the load versus displacement plot was constructed graphically as exactly one-sixth of a circle with radius equal to  $P_{\max}$  and with center on the  $P = 0$  line. Therefore, using plane geometry, the value of  $P_1$  shown in the figure could be calculated exactly to be  $0.910 P_{\max}$ . For this value, Eq. (1) holds with no approximation. In addition, values of  $P_1$  were calculated which correspond to five percent greater and less than the area under curve OAB. These values of  $P_1$  are shown as the dashed lines. By inspection of these dashed lines, it should be clear that the technique can be used to measure areas with better than five percent accuracy. In fact, by counting grid squares in the areas ACDA and BDDE, an accuracy of one or two percent can be obtained. The accuracy of the technique is also better for a load versus displacement plot with less deviation from linearity than that shown here.

#### REFERENCES

1. "Standard Test Method for  $J_{IC}$ , A Measure of Fracture Toughness, ASTM E-813," Annual Book of ASTM Standards, Vol. 03.01, ASTM, Philadelphia, PA, 1987, pp. 968-990.
2. Vossoughi, J., "A Simple, Quick, and Accurate Method to Measure the Area Under Curves," Experimental Techniques, Vol. 8, No. 10, 1984, pp. 26-27.



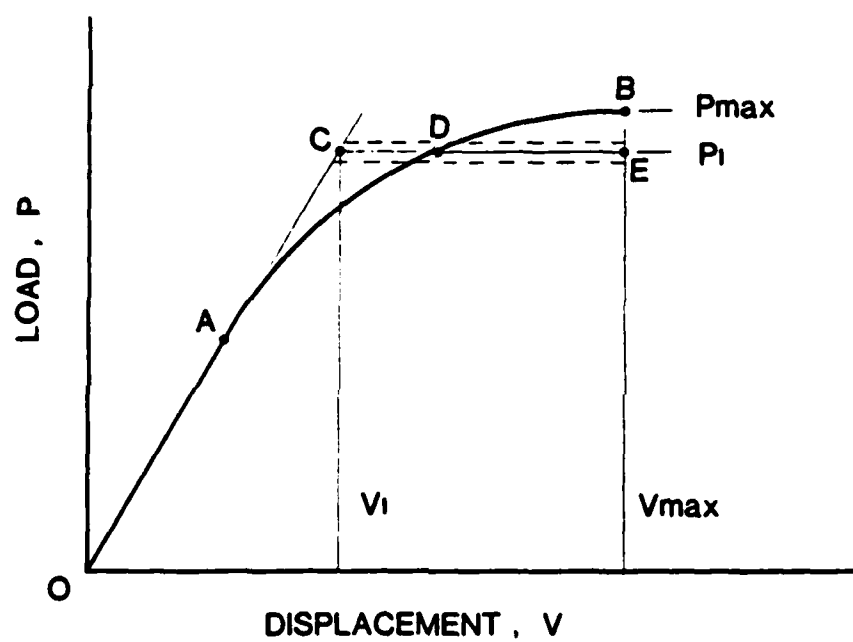


Figure 1. Sketch of load versus displacement showing a method for measuring area under the curve.

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